XI Caucasus Mathematic Olympiad
Maykop, March 13–18, 2026 year

Caucasus Mathematical Olympiad | Кавказская математическая олимпиада

Juniors. Day 1, March 14

1. 50 children are standing in a row. Determine if it is possible that among any 5 consecutive children the number of boys is less by 1 than the number of girls, and also among any 7 consecutive children the number of boys is less by 1 than the number of girls.

2. In a trapezoid $ABCD$ with $AD \parallel BC$ diagonals AC and BD are perpendicular. Let K be a point on the segment AD such that $AK + KC = AD + BC$. Prove that K is equidistant from A and C.

3. Find the smallest positive integer k such that the number 100^{100} can be represented as a product of 99 positive integers each of which is not greater than k.

4. Pasha and Vova play a game on 12×12 checkered board. They take turns, Pasha starts. On his turn, Pasha can choose two empty cells adjacent by side and place a coin heads-up in each of these two cells. Vova, on his turn, can choose any two diagonally adjacent cells and make tails-up each coin already placed in these two cells. The end of the game is announced immediately after Vova's move if Pasha cannot make his next move. Find the greatest possible number of heads-up coins, that Pasha can achieve at the end of the game, regardless of Vova's actions.

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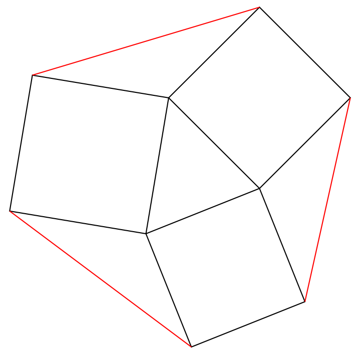
Seniors. Day 1, March 14

1. On the sides of a triangle whose area equals 1 squares are constructed outward. The ends of the sides of the squares sharing a vertex of the triangle are connected by red segments (see figure).

a) Prove that a triangle can be formed from three red segments; b) the area of this triangle equals 3.



2. Peter arranged n positive integers in a circle. For each integer, Basil calculated the product of that number and the GCD of the two numbers following it clockwise. All the numbers Basil calculated turned out to be equal. Are the original numbers necessarily equal, if a) $n = 15$; б) $n = 16$?

3. Positive real numbers a, b, c satisfy

$$\sqrt{a^2 + ab} + b + c = \sqrt{b^2 + bc} + c + a = \sqrt{c^2 + ca} + a + b.$$

Determine if it follows that $a = b = c$.

4. We are given a sequence $a_1, a_2, \ldots, a_n$ of positive integers. At each step, all numbers change simultaneously as follows: the i-th number is replaced by the number of indices j such that $a_j = a_i$, plus i. (For example the sequence: $1, 1, 4, 1, 4, 2, 1, 3$ becomes $5, 6, 5, 8, 7, 7, 11, 9$.) We then continue the process with the new sequence. Prove that after some time the sequence no longer changes.